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First order phase transition from free flow to synchronized flow in a cellular automata model

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Abstract. This paper presents an improved cellular automata model, which is based on three phase traffic theory and can reproduce the first order phase transition from free flow to synchronized flow. The fundamental diagram, the spacetime plots, and the 1-min average flux density diagram are presented. The autocorrelation and cross correlation functions are studied to identity the synchronized flow state. It is shown that the results of the model are well consistent with the empirical findings.

PACS. 45.70.Vn Granular models of complex systems; traffic flow – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 02.60.Cb Numerical simulation; solution of equations

1 Introduction

The systematic investigation of traffic flow has a quite long history (see, e.g., [1–10]). Recently, a more detailed analysis of empirical data has been given by Kerner and his colleague [8–10]. They pointed out that traffic flow can be either free or congested and the congested flow can be further distinguished into synchronized flow and wide moving jams. The complex spatio-temporal behavior of real traffic is due to the transitions between the three traffic phases. It is also pointed out that the phase transitions among the three phases are all first order.

Different explanations of these empirical findings and different models have been proposed by various groups in the last years [1–4]. Most model may be classified into the "fundamental diagram approach" since the steady state solutions of these models belong to a curve in the flow-density plane [11–13]. This curve which goes through the origin and has at least one maximum is called the fundamental diagram for traffic flow.

The fundamental diagram approach is successful in explaining several aspects of real traffic such as the forming of queue, the evacuation of jams and etc. However, as pointed out by Kener [4,14–17], the phase transitions and most of empirical spatial-temporal pattern features are qualitatively different from those which follow from the mathematical traffic flow models in the fundamental diagram approach.

Kerner introduced a cellular automata (CA) model based on the three-phase traffic theory which postulates that the steady states (homogeneous and stationary states, time-independent solutions in which all vehicles move with the same constant speed) of synchronized flow cover a two-dimensional region in the flow-density plane, i.e., there is no fundamental diagram of traffic flow [4,14–17]. The simulations show that the empirical spatial-temporal pattern features of traffic flow may be reproduced in this theory. The first-order phase transition from free flow to synchronized flow and the nucleation effect that governs this transition are also reproduced (see Fig. 8.3 in [4], Fig. 17b in [16]). Random time delays, probability, and the critical and threshold boundaries for this transition have been studied [4,16,17].

Recently, some new microscopic models based on three phase traffic theory have been developed, which can show the congested pattern features found by Kerner. For example, Lee et al. proposed a different CA model which also can describe the empirical spatial-temporal pattern features of traffic [18]. Besides, Jiang and Wu [19] presented a CA model based on the comfortable driving model of Knospe et al. [20], which can reproduce the synchronized flow quite satisfactorily.

In this paper, we focus on Jiang and Wu's model. From Figure 4 in [19], one can see that in this model, phase transition from free flow to synchronized flow is not correctly described: it is second order in the model. Therefore, the model needs to be improved to depict the empirical findings. This has been fulfilled by modifying the brake light rule. The modified model is able to reproduce the first order phase transition from free flow to synchronized flow.

The paper is organized as follows. In Section 2, the Jiang and Wu's model is briefly reviewed and the improved model are presented based on it. In Section 3, the simulation results are analyzed and compared with the empirical data. The conclusions are given in Section 4.

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2 Improved model

For the sake of the completeness, we briefly recall Jiang and Wu's model. The parallel update rules of the model are as follows:

1. Determination of the randomization parameter $p_n(t+1)$:
   $p_n(t+1) = p(v_n(t), b_{n+1}(t), t_{h,n}, t_{s,n})$.
2. Acceleration:
   if $((b_{n+1}(t) = 0$ or $t_{h,n} \geq t_{s,n})$ and$(v_n(t) > 0))$ then:
   $v_n(t+1) = \min(v_n(t) + 2, v_{max})$
   else if $(v_n(t) = 0)$ then:
   $v_n(t+1) = \min(v_n(t) + 1, v_{max})$
   else:
   $v_n(t+1) = v_n(t)$.
3. Braking rule:
   $v_n(t+1) = \min(d_n^{eff}, v_n(t+1))$.
4. Randomization and braking:
   if $(rand() < p_n(t+1))$ then: $v_n(t+1) = \max(v_n(t+1) - 1, 0)$.
5. The determination of $b_n(t+1)$:
   if $(v_n(t+1) < v_n(t))$ then: $b_n(t+1) = 1$
   if $(v_n(t+1) > v_n(t))$ then: $b_n(t+1) = 0$
   if $(v_n(t+1) = v_n(t))$ then: $b_n(t+1) = b_n(t)$.
6. The determination of $t_{st,n}$:
   if $v_n(t+1) = 0$ then: $t_{st,n} = t_{st,n} + 1$
   if $v_n(t+1) > 0$ then: $t_{st,n} = 0$.
7. Car motion:
   $x_n(t+1) = x_n(t) + v_n(t+1)$.

Here x_n and v_n are the position and velocity of vehicle n (here vehicle $n+1$ precedes vehicle n), d_n is the gap of the vehicle n, b_n is the status of the brake light (on(off)$\rightarrow b_n = 1(0)$). The two times $t_{h,n} = d_n/v_n(t)$ and $t_{s,n} = \min(v_n(t), h)$, where h determines the range of interaction with the brake light, are introduced to compare the time $t_{h,n}$ needed to reach the position of the leading vehicle with a velocity dependent interaction horizon $t_{s,n}$. $d_n^{eff} = d_n + \max(v_{anti} - gap_{safety}, 0)$ is the effective distance, where $v_{anti} = \min(d_{n+1}, v_{n+1})$ is the expected velocity of the preceding vehicle in the next time step and gap_{safety} controls the effectiveness of the anticipation. $rand()$ is a random number between 0 and 1, $t_{st,n}$ denotes the time that the car n stops. The randomization parameter p is defined:

$$p(v_n(t), b_{n+1}(t), t_{h,n}, t_{s,n}) = \begin{cases} p_b: & \text{if } b_{n+1} = 1 \quad \text{and } t_{h,n} < t_{s,n} \\ p_0: & \text{if } v_n = 0 \quad \text{and } t_{st,n} \geq t_c \\ p_d: & \text{in all other cases.} \end{cases}$$

Here t_c is a parameter.

In what follows, the model is improved to reproduce the first order phase transition from free flow to synchronized flow. To this end, a new variable $t_{f,n}$ is introduced. It denotes the time that car n is in the state $v_n \geq v_c$. We suppose that if $v_n(t+1) \geq v_c$ and $t_{f,n} \geq t_{c1}$, then $b_n(t+1)$ remains to be zero despite of the value of $v_n(t)$. For the determination of $t_{f,n}$, it is simply that

$$\text{if } v_n(t+1) \geq v_c \text{ then: } t_{f,n} = t_{f,n} + 1$$

$$\text{if } v_n(t+1) < v_c \text{ then: } t_{f,n} = 0.$$

Here v_c and t_{c1} are parameters.

In the next section, the simulations are carried out. In the simulations, the parameter values are $t_c = 10$, $t_{c1} = 30$, $v_c = 18$, $v_{max} = 20$, $p_d = 0.1$, $p_b = 0.94$, $p_0 = 0.5$, $h = 6$, $gap_{safety} = 7$. Each cell corresponds to 1.5 m and a vehicle has a length of five cells. One time step corresponds to 1 s. The periodic boundary condition is used and the system size is $L = 10\,000$.

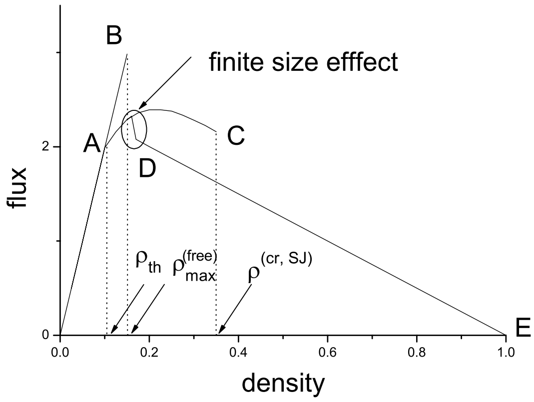

Fig. 1. The fundamental diagram of the improved model. This diagram is associated with 2Z-characteristics for the first order transition from free flow to synchronized flow and the first order transition from synchronized flow to jam in speed density plane (see, e.g., Fig. 17.10 in [4]), line DE is associated with the line J discussed in detailed in the book [4].

3 Simulation results

In this section, the simulation results are presented. In Figure 1, we show the fundamental diagram of the improved model. The flux is calculated from $J = \rho\langle v\rangle$, where $\langle v\rangle$ is average velocity of the vehicles. When the density $\rho < \rho_{th}$, the system is in free flow. When $\rho > \rho_{th}$, one can see that three branches are distinguished. The branch AB starts from the initial condition I_1: the cars are distributed homogeneously and $v_n > v_c$ and $t_{f,n} > t_{c1}$ for all cars. For the case, the free flow is maintained (see Fig. 2a). The branch AC starts from the initial condition I_2: the cars are distributed homogeneously and $v_n < v_c$ and $t_{f,n} = 0$ for all cars. For the case, the simulations show that $t_{f,n} > t_{c1}$ can never be reached for any car. Thus the improved model reduces to the original model. The homogeneous distribution of cars leads to light synchronized flow or heavy synchronized flow (see Figs. 5c and d in Ref. [19]). The branch DE starts from the megajam. For the case, the system is the coexistence of jam, free flow and light synchronized flow (see Figs. 5e and f in Ref. [19]).

For the density $\rho > \rho_{max}^{(free)}$, even if starting from the initial condition I_1, the free flow cannnot be maintained. It

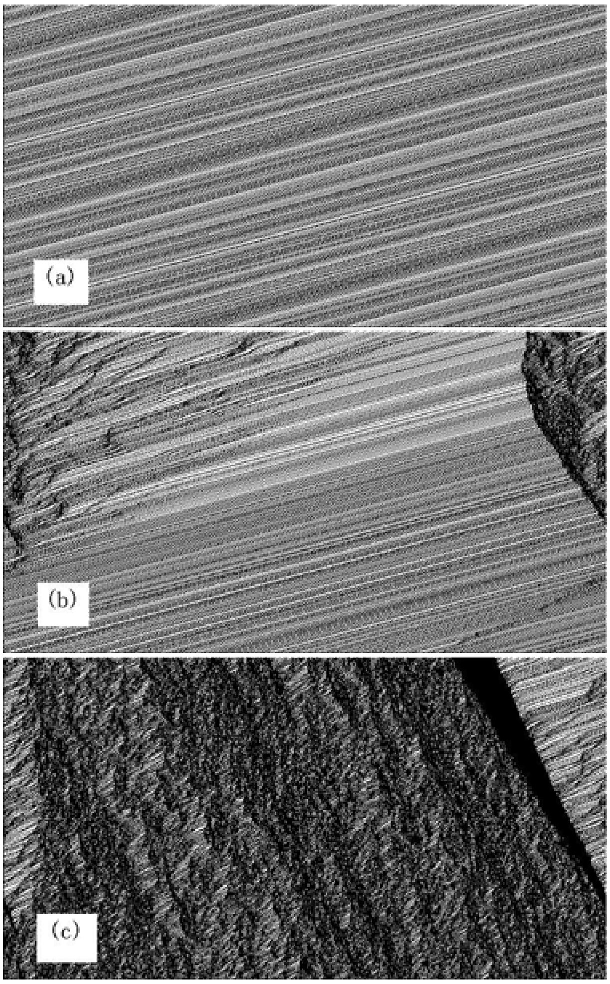

Fig. 2. (a) The free flow phase starting from initial condition I_1, $\rho = 0.15$; (b) the free flow spontaneously evolves into the synchronized flow, $\rho = 0.16$; (c) the synchronized flow spontaneously evolves into the jam, $\rho = 0.37$.

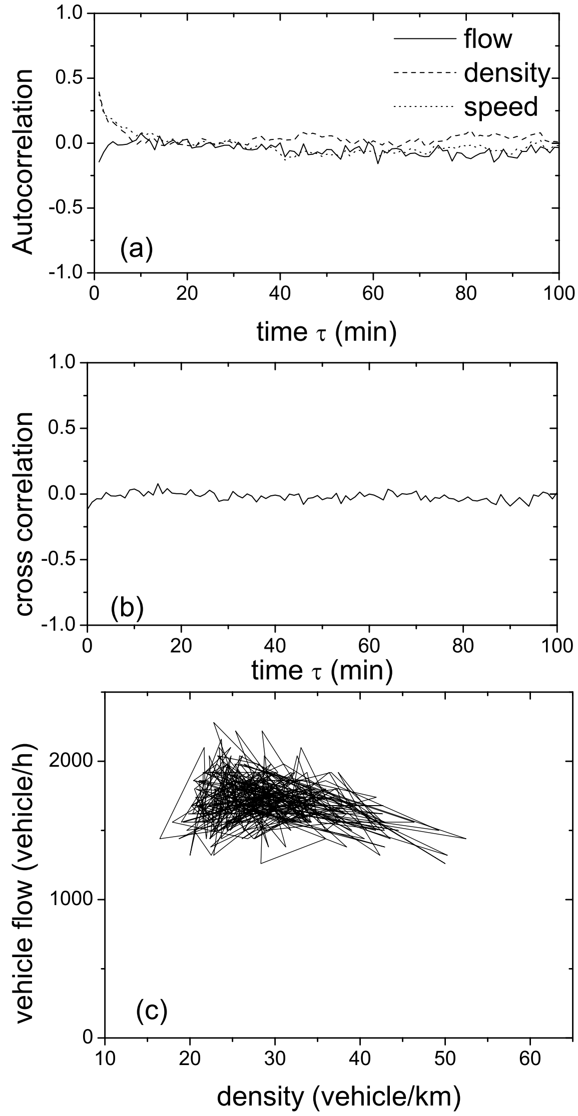

Fig. 3. (a) Autocorrelation function and (b) cross correlation between density and flow, of the synchronized flow. (c) The 1-min averaged flux density diagram corresponding to the synchronized flow in (a) and (b).

will spontaneously evolve into the synchronized flow after certain time (see Fig. 2b) [14]. Similarly, for the density $\rho > \rho^{(cr,SJ)}$, even if starting from the initial condition I_2, the synchronized flow cannot be maintained. It will spontaneously evolve into the coexistence of jam, free flow and light synchronized flow after certain time (see Fig. 2c) [14].

In order to identify more clearly the synchronized traffic, the correlation function is investigated. First we consider the autocorrelation

$$a_x(\tau) = \frac{\langle x(t)x(t+\tau)\rangle - \langle x(t)\rangle^2}{\langle x^2(t)\rangle - \langle x(t)\rangle^2}$$

of the aggregated quantities $x(t)$. In Figure 3a, the autocorrelations of one-minute aggregates of the density, flow, and average speed of a synchronized state are shown. One can see that there is no correlations on time scales larger than 1 min. Figure 3b shows that the cross correlation

$$c_{x,y}(\tau) = \frac{\langle x(t)y(t+\tau)\rangle - \langle x(t)\rangle\langle y(t)\rangle}{\sqrt{\langle x^2(t)\rangle - \langle x(t)\rangle^2}\sqrt{\langle y^2(t)\rangle - \langle y(t)\rangle^2}}$$

between density and flow vanishes. Both clarify the existence of the synchronized flow state.

Next we study the phase transition between the free flow and the synchronized flow. We slowly add cars to a homogeneous free flow, the traffic system evolves along the branch AB (Fig. 4). However, when the system density exceeds $\rho_{max}^{(free)}$, the free flow can only exist for a finite time, then the synchronized flow appears spontaneously. The flow rate drops to a lower value. Then we slowly remove the cars from the system. The system evolves along the branch CA instead of returning to the branch AB. This obviously is a first order phase transition.

In Figure 5, we show the 1-min average flux density diagram. One can see that it is in consistent with the

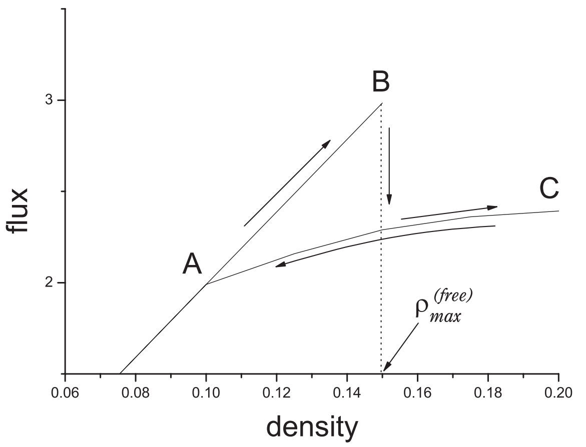

Fig. 4. The phase transition from free flow to synchronized flow when adding cars slowly to a free flow system, and the transition from synchronized flow to free flow when removing cars from synchronized traffic. This curve is associated with the Z-characteristic for the first order transition from free flow to synchronized flow in speed density plane (see e.g., Fig. 17.3 in [4]).

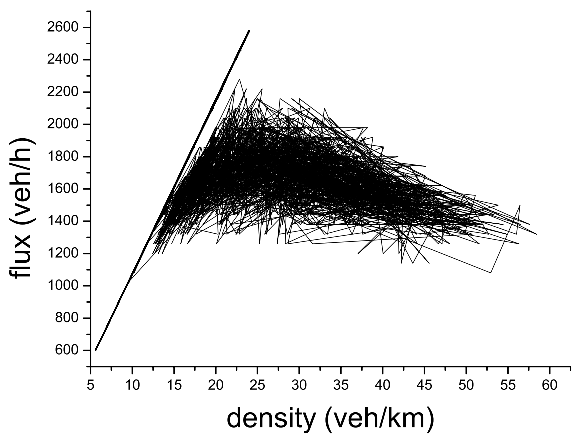

Fig. 5. The 1-min averaged flux density diagram.

empirical observations (cf. Fig. 1 in Ref. [8][1]). The first order phase transition from free flow to synchronized flow as well as the wide scatter of the synchronized flow data is reproduced [17].

We investigate the effect of t_{c1} on the fundamental diagram. The simulations show that with the increase of t_{c1}, $\rho_{max}^{(free)}$ decreases while $\rho^{(cr,SJ)}$ remains unaltered. For a large enough t_{c1}, the improved model reduces to the original one of Jiang and Wu's. However, with the decrease of t_{c1}, $\rho_{max}^{(free)}$ increases and $\rho^{(cr,SJ)}$ decreases. For a small t_{c1}, the synchronized flow cannot be reproduced and the improved model reduces to a model similar to the velocity dependent randomization (VDR) model.

[1] We believe that the much lower flow rate in reference [8] is caused by mixture with slow vehicles, see also reference [21].

4 Conclusions

This paper presented an improved CA model which can reproduce the first order phase transition from the free flow to synchronized flow. The fundamental diagram are analyzed: it is shown that three branches can be classified. The spacetime plots of different phases as well as the transition between different phases are shown. The autocorrelation and cross correlation functions of the synchronized flow state are investigated. The 1-min average flux density diagram is presented and it is satisfactorily consistent with the empirical findings. The effect of the parameter t_{c1} is investigated, and it is found that the improved model may reduce to either the original model of Jiang and Wu's or the VDR model depending on the value of t_{c1}.

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